

8.4 The Laws of Logarithms

Exponential Form

$$y = b^x$$

Logarithmic Form

$$x = \log_b y$$

Properties of Logarithms

$$\log_b b^n = n$$

$$b^{\log_b n} = n$$

$$\log_b 1 = 0$$

Power Law for Logarithms

$$\log_b x^n = n \log_b x \quad x > 0, b > 0, b \neq 1,$$

Product Law for Logarithms

$$\log_b(mn) = \log_b m + \log_b n \quad b > 0, b \neq 1, m > 0, n > 0$$

Quotient Law for Logarithms

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n \quad b > 0, b \neq 1, m > 0, n > 0$$

Change of Base Formula

$$\log_b x = \frac{\log x}{\log b} \quad x > 0, b > 0, b \neq 1$$

Example 1: Applying the Power Law of Log

Evaluate.

a) $\log_3 9^4$

b) $\log_2 8^5$

c) $\log 0.001^2$

d) $\log_5 (\sqrt{125})^3$

Example 2: Applying the Product/Quotient Law of Log

Write as a single logarithm. State any restrictions.

a) $\log_3 6 + \log_3 8 - \log_3 12$

b) $\log x + \log y + \log 2x - \log y^2$

Example 3: Evaluate Using the Laws of Log

Evaluate.

a) $\log_8 4 + \log_8 16$

b) $\log_3 486 - \log_3 6$

c) $2\log 5 + \frac{1}{2}\log 16$

Example 4: Write the Log of a Product/Quotient as a Sum/Difference of Log

Write as a sum or difference of logarithms. Simplify, if possible. State any restrictions.

a) $\log_4(ab)$

b) $\log_3(ab^2c)$

c) $\log(20)$

d) $\log\left(\frac{ab}{\sqrt{c}}\right)$

Example 5: Simplify Algebraic Expressions

Simplify. State any restrictions.

a) $\log(\sqrt{x})^3 + \log x^2 - \log \sqrt{x}$

b) $\log(2x-2) - \log(x^2-1)$

Example 6: Applying the Change of Base Formula

Evaluate to 4 decimal places.

a) $\log_3 17$

b) $\log_{\frac{1}{2}} 10$

Write as a single logarithm

a) $\frac{\log 5}{\log 7}$

b) $\frac{\log(x+1)}{\log(2x)}$

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